

$\mathbb{Z}_3 \times \mathbb{Z}_3$ acts ^{non-trivial} on \mathbb{Z}_7
 or
 \mathbb{Z}_9 " " \mathbb{Z}_7

G acts on H as a group :

$$G \times H \rightarrow H$$

$$(g, h) \mapsto h^g$$

- $(h^{g_2})^{g_1} = h^{(g_1 g_2)}$

- $h^{e_G} = h$

- $(h_1 \cdot h_2)^g = (h_1^g \cdot h_2^g)$

$$(g \cdot h)$$

$$(g_1 (g_2 \cdot h)) = (g_1 g_2) \cdot h$$

$$(e_G \cdot h = h)$$

$$\times$$

or

$$\exists \text{ homo. } \rho: G \rightarrow \text{Aut}(H)$$

\Rightarrow

$$H \cong_p G$$

Fact: If ρ is non-trivial

$H \cong_p G$ non-abelian.

$$H \rtimes_f G \stackrel{\text{set}}{=} H \times G$$

$$(h_1, g_1) \cdot (h_2, g_2) = \cancel{(h_1 h_2, g_1 g_2)}$$

$$\searrow$$

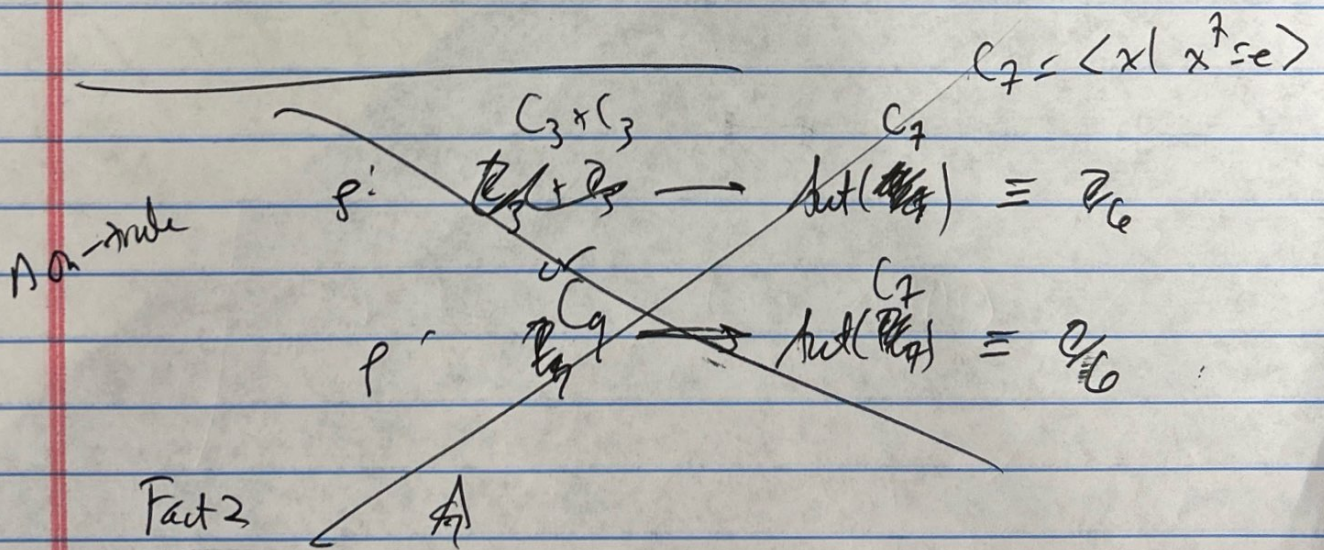
$$(h_1 h_2^{g_1}, g_1 g_2)$$

$$h_2^{g_1} := f(g_1)(h_2)$$

$$f \text{ non-trivial} \Rightarrow \exists g, h \text{ s.t. } h^g \neq h$$

$$(e_H, g) \cdot (h, e_G) = (e_H \cdot h^g, g \cdot e_G) = (h^g, g)$$

$$(h, e_G) \cdot (e_H, g) = (h, g) \neq$$

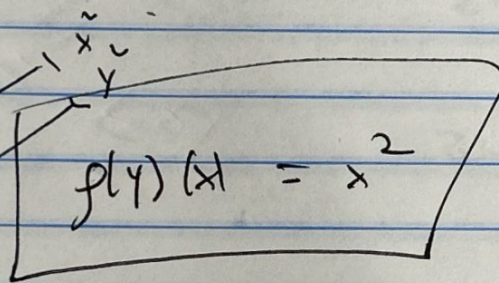


$$C_3 \times C_3 = \langle a, b \mid a^3 = 1, b^3 = 1, ab = ba \rangle$$



$$2(b) \quad \text{UMP } \langle \tilde{x}, \tilde{y} \mid \tilde{x}^2 = 1, \tilde{y}^4 = 1, \tilde{y}\tilde{x}\tilde{y}^{-1} = \tilde{x}^2 \rangle$$

$$C_7 \times C_9$$



$$\tilde{y}\tilde{x} = \tilde{x}^2\tilde{y}$$

$$(x, 1)$$

$$(1, y)$$

- Map exists by UMP
- onto : not hard
- ETS: #L ≤ 63 ✓

✓ Fact 2 $\text{Aut}(\mathbb{Z}_p) \cong (\mathbb{Z}_p)^\times = (\mathbb{Z}_p - \{0\}, \cdot)$
 $(m \mapsto n \cdot m) \leftrightarrow n$

Fact 3 $(\mathbb{Z}_p)^\times$ is cyclic

~~$(\mathbb{Z}_7)^\times = \langle 2 \rangle$~~
 $= \langle 3 \rangle$

~~$2 \mapsto 2^2 \mapsto 2^3$~~
 8
 $3 \mapsto 3^2 \mapsto 18$
 $2 \mapsto 6 \mapsto 12$
 $4 \mapsto 13$
 $5 \mapsto 15$
 $6 \mapsto 10$
 $7 \mapsto 11$

$f: C_3 \times C_3 \rightarrow \text{Aut}(C_7) \cong \mathbb{Z}_6$
 C_9

$C_7 = \langle x \mid x^7 = 1 \rangle$

$\text{Aut}(C_7) \cong \alpha \quad \alpha(x) = x^3$

$\langle \alpha \rangle = \text{Aut}(C_7)$

$C_9 = \langle y \mid y^9 = 1 \rangle$

$C_7 = \langle x \mid x^7 = 1 \rangle$

$f(y) = \alpha \in \text{Aut}(C_7)$

$\alpha(x) = x^2$
 $|A| = 3$